Exam Seat No:________________ Enrollment No:____________________

C.U.SHAH UNIVERSITY

Wadhwan City

Subject Code : 5SC01MTC4 **Summer Examination-2014** **Date: 19/06/2014**

Subject Name Topology-I

Branch/Semester:- M.Sc(Maths)/I **Time:10:30 To 1:30**

Examination : Remedial

Instructions:-

(1) Attempt all Questions of both sections in same answer book / Supplementary

(2) Use of Programmable calculator & any other electronic instrument is prohibited.

(3) Instructions written on main answer Book are strictly to be obeyed.

(4) Draw neat diagrams & figures (If necessary) at right places

(5) Assume suitable & Perfect data if needed

SECTION-I

Q-1 a) Which of the following are open in standard topology? (02)

(i) $(a,\ b)$ (ii) $(a,\ b]$ (iii) $[a,\ b)$ (iv) $[a,\ b]$

b) Let $X = \{a, b, c, d\}$. Check which of the following are topologies on X ? (02)

(i) $\tau_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}\}$

(ii) $\tau_2 = \{X, \{a\}, \{a, b\}\}$

c) Which topology is stronger, standard topology on R or lower limit topology on R ? (01)

d) Define topology. (01)

e) Define open basis for a topological space. (01)

Q-2 a) Define co-countable topology on R. Show that it is a topology on R. (05)

b) Let β be a basis for a topology on X. Define (05)

$$\tau = \{U \subset X: \forall x \in U, there\ exists\ B \in \beta\ such\ that\ x \in B \subset U\}.$$

Then Show that τ is a topology on X.

c) Let X be a non-empty set. For $x, y \in X$ define (04)

$$d(x,y) = \begin{cases} 1 & if\ x \neq y \\ 0 & if\ x = y \end{cases}$$

Show that $(X,\ d)$ is a metric space.

OR

Q-2 a) Let $(X,\ d)$ be metric space. For $x, y \in X$, define $d'(x,\ y) = \frac{d(x,y)}{1+d(x,y)}$. Show that d' is a metric on X. (05)

b) Define $\tau = \left\{ \begin{matrix} \text{U} \subset \text{R: for every x} \in \text{U there exists an open interval} \\ (a,\ b)\ such\ that\ x \in (a,\ b) \subset U \end{matrix} \right\}$. Is this a topology on R ? Justify your answer. (05)

c) Let X be a topological space and $A \subset X$. Suppose that for each $x \in A$ there is an open set U in X such that $x \in U \subset A$. Show that A is open in X. (04)

Q-3 a) Let $(X_1, \tau_1), (X_2, \tau_2), \dots (X_m, \tau_m)$ be topological spaces. Let $X = X_1 \times X_2 \times \dots \times X_m = \prod_{i=1}^{m} X_i$, and $\beta = \{\prod_{i=1}^{m} U_i : U_i \in \tau_i, i = 1,2, \dots m.\}$. Show that β is a basis for some topology on X. (05)

b) Let Y be a subspace of X. Prove that a set A is closed in Y if and only if $A = B \cap Y$ for some closed set Bin Y. (05)

c) Let X be a topological space and $A \subset X$. Prove that $\bar{A} = A \cup A'$. (04)

OR

Q-3 a) Let X and Y be topological spaces and $f: X \to Y$ be a function show that the following are equivalent. (05)

(i) f is continuous.

(ii) f is continuous at each point of X.

b) Let X be a topological space and $A \subset X$. Prove that $Bd(A) = \bar{A} - A^{\circ}$. (05)

c) Is $\overline{A \cap B} = \bar{A} \cap \bar{B}$ in any topological space ? Justify your answer. (04)

SECTION-II

Q-4 a) Define homeomorphism. (01)

b) Is discrete topological space a T_1 space ? (01)

c) Is lower limit topological space a T_2 space ? (01)

d) Define locally compact space. (01)

e) State Tietze Extension Theorem. (02)

f) Define locally connected space. (01)

Q-5 a) Let X be a topological space and $A \subset X$. Let (x_n) be a sequence in A such that $x_n \to x$, then prove that $x \in \bar{A}$. (07)

b) Let X be a topological space. Prove that X is a T_2 space if and only if $\{x\} = \cap \{\bar{U} : U \text{ is a neighbourhood of } x, \forall x \in X\}$. (07)

OR

Q-5 a) Prove that continuous image of a compact space is compact. (07)

b) Prove that every closed and bounded interval in R is compact. (07)

Q-6 a) Prove that every metric space has its completion. (07)

b) Prove that every compact T_2 space (Hausdorff space) is T_3 sapce (regular). (07)

OR

Q-6 a) State and prove Urysohn's lemma. (10)

b) Prove that the product of two connected spaces is a connected space. (04)

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